Turbofan

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Introduction

The turbo fan. I am sure most people are aware of what a plane is. It must have been one of the greatest engineering accomplishments ever created, and it all started with the Wright Brothers. Fast forward to today's day and age and we now have aircrafts that can transport hundreds of people over hundreds of miles in the air. There is no supernatural phenomenon that makes us fly but we do and that is because of engineering. Most of the credit comes from the engines that power the airplane. These big engines are what propel the plane and keep it going at such high speeds for such a long time. These big engines that you see hanging on the wings of a plane are turbo fans. They provide the energy for flight. They are a part of why airplanes are so special, but how exactly are they created? Most likely, a long time ago, it was just trial and error, but we have grown as engineers and have created a way to model such turbo fans with calculations that can go beyond what we are imagining. We base what we can turn into a reality from what we calculate. The theory is all here. We have a system of components working together that are all based on a law that was created so long ago, Newtons first. This is the basis of how we make such calculations model real life scenarios. I will take you through the process of how the calculations for a turbofan work in hope of creating the one that is most efficient. This will be done with Cold air standard assumptions in mind and with the use of MATLAB.

Theory

Let us begin with the theory of all the components that we will be using to create the turbofan. This consists of a system with a diffuser, two nozzles, one fan, one compressor, one combustion chamber, and two turbines. Figure one below shows the turbofan we will be performing calculations on.

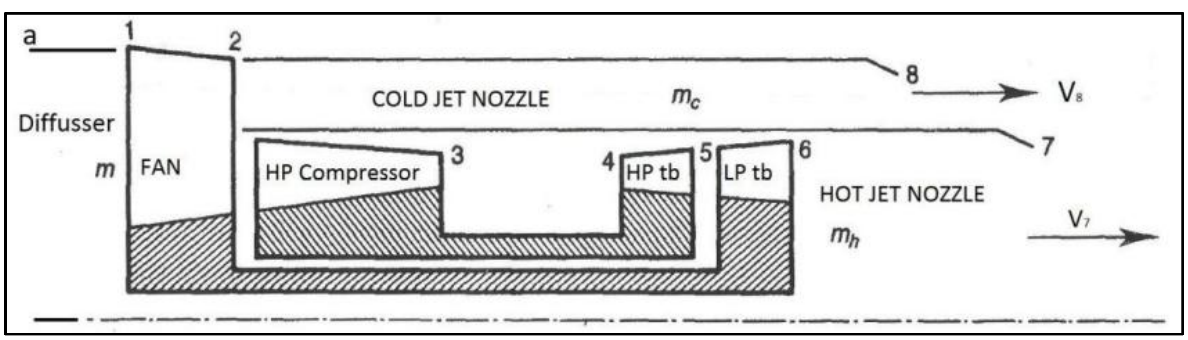


Figure 1: Twin-spool turbofan engine

Diffuser

The first component in the system is the one at the beginning, the diffuser. This is seen at the front of those big engines on airplanes and is simply a truncated cone with the smaller area of the cone being the inlet and the exit being the bigger area. To explain the theory behind the calculations for the diffuser, we can always start with the first law.

$$\dot{Q}_{net} - \dot{W}_{net} + \sum \dot{m}_i \left(h_i + \frac{1}{2} * v_i^2 + gz_i\right) - \sum \dot{m}_e \left(h_e + \frac{1}{2} * v_e^2 + gz_e\right) = 0$$

Now we can start removing terms that will not interact with the properties of the diffuser. The first two are work and heat which are both not being added or removed. Remember, its just air going through a truncated cone. Next is the mass flow rate. Since no mass is getting added or removed, the inlet and exit have the same mass flow rate. Next is potential energy and in this case the inlet and exit remain on the same level so there is no change in Z, therefore removing that term which leaves us with the following:

$$h_a + \frac{1}{2} * v_a^2 = h_1 + \frac{1}{2} * v_1^2$$

The next step before simplifying and modifying the equation would be to consider the units present and enthalpy is in kJ/kg while the velocity is in m^2/s^2 which converts directly to J/kg so to fix this, we will divide the velocity term by 1000, matching the units.

$$h_a + \frac{1}{2000} * v_a^2 = h_1 + \frac{1}{2000} * v_1^2$$

Now that we have the bulk of the equation, we will now transform the changes in enthalpy with cold air standard assumptions. This directly relates the changes in enthalpy to the temperatures at the respective states that way we don’t have to look in the tables.

$$C_p(T_1 - T_a) = h_1 - h_a$$

$$\frac{1}{2000} * v_a^2 - \frac{1}{2000} * v_1^2 = C_p(T_1 - T_a)$$

Still in this equation we have two unknowns which would be the temperature at state one and the velocity at state one but with the idea of the mass flow rates being the same on the inlet and exit, it's possible to relate the velocity and temperature. This will also include the utilization of one more formula regarding the isentropic properties of a component.

$$\frac{T_1}{T_a} = \left(\frac{\nu_a}{\nu_1}\right)^{k-1}; \quad \dot{m} = \frac{A_a * v_a}{\nu_a} = \frac{A_1 * v_1}{\nu_1}$$

$$v_1 = \left(\frac{A_a}{A_1}\right)\left(\frac{\nu_1}{\nu_a}\right) * v_a$$

$$\frac{T_1}{T_a} = \left(\frac{\nu_a}{\nu_1}\right)^{k-1} \rightarrow \left(\frac{T_1}{T_a}\right)^{\frac{1}{k-1}} = \frac{\nu_a}{\nu_1} \rightarrow \frac{1}{\frac{\nu_a}{\nu_1}} = \frac{1}{\left(\frac{T_1}{T_a}\right)^{\frac{1}{k-1}}} \rightarrow \frac{\nu_1}{\nu_a} = \left(\frac{T_a}{T_1}\right)^{\frac{1}{k-1}}$$

$$v_1 = \left(\frac{A_a}{A_1}\right)\left(\frac{T_a}{T_1}\right)^{\frac{1}{k-1}} * v_a$$

$$\frac{1}{2000} * {v_a}^2 - \frac{1}{2000} * \left(\frac{A_a}{A_1}\left(\frac{T_a}{T_1}\right)^{\frac{1}{k-1}} v_a\right)^2 = C_p(T_1 - T_a)$$

Now the equation is complete. We have only one unknown which is the temperature at 1 and although we do not have any information on the actual size of the inlet and exit of the diffuser, we do have the ratio of the area at the exit to the area at the inlet. This is referred to as DAR in my code as diffuser area ratio and it is A1/Aa so the inverse must be taken but now all that is left is to solve for T1 which can be seen in the MATLAB code however, every calculation has been made with the fact that the diffuser is a perfect isentropic component but that is not the case. It does have an isentropic efficiency so we will have to apply that at the end of our calculations and instead of referring to the previous T1 as the true T1, we will call it T1s, the ideal T1. The true T1 is greater than the ideal because the entropy does change a little were friction occurs along the walls of the diffuser and increases the temperature.

$$\frac{1}{2000} * {v_a}^2 - \frac{1}{2000} * \left(\frac{A_a}{A_1}\left(\frac{T_a}{T_{1s}}\right)^{\frac{1}{k-1}} v_a\right)^2 = C_p(T_{1s} - T_a); \quad \eta_d = \frac{T_{1s}}{T_1}$$

Fan (Compressor):

The big fan that you see at the front of the jet is this exact component. Although it is called a fan, it performed the same functions as a compressor which would be to increase pressure. Ideally this component also is isentropic, meaning we can use ideal-gas and isentropic equations with the information that we are given but to power this compressor is what we will use the first law for. We will form an equation connecting two components which you can see in figure one as the low-pressure turbine and fan. Below is the ideal isentropic equation we will use to determine the temperature at 2 as well as its isentropic efficiency equation because in reality, it's not ideal.

$$\left(\frac{T_{2s}}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \quad ; \quad \eta_f = \frac{T_{2s} - T_1}{T_2 - T_1}$$

In this equation, the pressure ratio of the fan is to be chosen by us and the temperature at state 1 is one that was previously solved for in the diffuser equations. Also with a clear pressure ratio, it takes a simple multiplication of the pressure ratio and P1 to find P2, so $P_2 = \frac{P_2}{P_1} * P_1$. Like I stated before, later on we will have to connect this component to the low-pressure turbine as that is how the fan gets the work that it needs.

High-Pressure Compressor:

The compressor follows the same equations as the fan as it is the same component that performs the same function, increasing the pressure. The high-pressure compressor also must receive work from somewhere but this time it's not from the low-pressure turbine. Instead, it draws its' work from the high-pressure turbine and an equation like the one for the fan and low-pressure turbine will be drawn later on. We must also keep in mind the mass flow rates that affect such components and, in this scenario, the only one going through the high-pressure turbine and high-pressure compressor is the hot air mass flow rate. Below are the equations we will use to find the temperature and pressure. Keep in mind, this component is not ideal, but an ideal scenario is needed to perform isentropic efficiency calculations.

$$\left(\frac{T_{3s}}{T_2}\right) = \left(\frac{P_3}{P_2}\right)^{\frac{k-1}{k}} \quad ; \quad \eta_{HPC} = \frac{T_{3s} - T_2}{T_3 - T_2}; \quad P_3 = \frac{P_3}{P_1} * P_1; \quad \frac{P_3}{P_2} = \frac{\frac{P_3}{P_1}}{\frac{P_2}{P_1}}$$

I used the overall pressure ratio from state one to three in order to find the pressure at 3 and all the other information for the ideal case like the pressure ratio from state 3 to state 2 can be found with the equation shown on the right. K is also known so the ideal temperature at three can be found

Combustion Chamber:

A combustion chamber is where the air and fuel get mixed together and ignite but in simple cycle models, we can act as if that ignition and combustion is just heat getting added to the air. In reality the combustion chamber is a just a chamber or tube that the air travels through while heat is being added to it. With no chang in the dimensions of the tube from three to four, we can assume that ideally, the pressure will not change between the two states, but of course since heat is added, the temperature will rise and that is a variable we are allowed to change so it's a known variable. To find the heat added, we can refer back to the first law and derive our equation from there.

$$\dot{Q}_{net} + \sum \dot{m}_i h_i - \sum \dot{m}_e h_e = 0$$

$$P_4 = P_3$$

I removed the work because no work is going in or out of the mixing chamber and as for the potential and kinetic energy, there is no change in that because the air moves through the same tube that is at the same height on both ends. To finish the equation and simplify we can plug in the states which we are analyzing as well as apply our standard cold air assumptions.

$$\dot{Q}_{in} = (h_4 - h_3) * \dot{m}_h \rightarrow \dot{Q}_{in} = C_p(T_4 - T_3) * \dot{m}_h$$

High-Pressure Turbine:

The turbines' goal and purpose are to expand the air and heat that is at state four and propel that to the next state. It will lower the pressure and temperature but to derive the equation for which we can obtain the temperature and pressure, we can look at the component that it's connected to. In figure 1, it's clear that the high-pressure turbine is connected to the high-pressure compressor and we can use Newton's first law to go from there.

$$\dot{W}_{out,HPT} = \dot{m}_h(h_4 - h_5)$$

With the derivation of the work that the HPT produces we can now relate it to the component that it gives its work to and that is the high-pressure compressor as seen in figure one. The work it receives is in the equation below as well as its relation to the turbine.

$$\dot{W}_{in,HPC} = \dot{m}_h(h_3 - h_2)$$

$$\dot{W}_{out,HPT} = \dot{W}_{in,HPC}$$

$$\dot{m}_h(h_4 - h_5) = \dot{m}_h(h_3 - h_2)$$

The equation is complete, and we have the ideal process for the turbine but like the other components, it also has an isentropic efficiency that can be applied after finding the ideal properties. Below is the equation after converting using cold air standard assumptions as well as the application of the efficiency for the turbine.

$$\dot{m}_h C_p(T_4 - T_5) = \dot{m}_h C_p(T_3 - T_2)$$

$$T_4 - T_5 = T_3 - T_2; \qquad \eta_{HPT} = \frac{T_4 - T_5}{T_4 - T_{5s}}$$

Now that we have the temperature of an ideal process, we can resort back to using the isentropic equation of pressure ratios to find the pressure at state five.

$$\left(\frac{T_{5s}}{T_4}\right) = \left(\frac{P_5}{P_4}\right)^{\frac{k-1}{k}}$$

Low-Pressure Turbine:

The next component is the low-pressure turbine and performs the same function as the high-pressure turbine, but it gives its work to a different component and that is the big fan at the front. By using the first, law we can derive an equation for this component as well, however, it's important to keep in mind the mass flow rates that encounter each of the components which will be shown in the derivation.

$$\dot{W}_{out,LPT} = \dot{m}_h(h_5 - h_6)$$

$$\dot{W}_{in,f} = \dot{m}(h_2 - h_1)$$

$$\dot{W}_{out,LPT} = \dot{W}_{in,f}$$

$$\dot{m}_h(h_5 - h_6) = \dot{m}(h_2 - h_1)$$

Now we can apply our cold air standard assumption as well relate the hot mass flow rate to total mass flow rate as they create an equation with three unknowns but we do know the back pressure ratio so we can use that. After we can relate those, all that is to apply its isentropic efficiency and find the pressure at state six like we did for five.

$$\dot{m}_h C_p(T_5 - T_6) = \dot{m}C_p(T_2 - T_1) \rightarrow \dot{m}_h(T_5 - T_6) = \dot{m}(T_2 - T_1)$$

$$\dot{m}_c + \dot{m}_h = \dot{m}$$

$$[\dot{m}_h(T_5 - T_6) = (\dot{m}_c + \dot{m}_h)(T_2 - T_1)]\frac{1}{\dot{m}_h}$$

$$(T_5 - T_6) = \left(\frac{\dot{m}_c + \dot{m}_h}{\dot{m}_h}\right)(T_2 - T_1)$$

$$(T_5 - T_6) = \left(\frac{\dot{m}_c}{\dot{m}_h} + 1\right)(T_2 - T_1); \qquad \eta_{LPT} = \frac{T_5 - T_6}{T_5 - T_{6s}}$$

$$\left(\frac{T_{6s}}{T_5}\right) = \left(\frac{P_6}{P_5}\right)^{\frac{k-1}{k}}$$

Hot Nozzle:

This is the end component of the engine. After all the internal components of the engine have brought the air to a certain temperature and pressure, we can now derive an equation as to how the properties of the air are at the end which can help us determine the speed of air and mass flow rates but more on that later. Starting with heat and work, no work or heat is being added and the only internal energy components changing are enthalpy and kinetic energy which leaves us with the following.

$$\dot{m}_h(h_6) = \dot{m}_h\left(h_{7s} + \frac{1}{2000} * {v_{7s}}^2\right); \quad assumption: v_6 \ll v_7$$

In this equation we are assuming that the velocity at the exit is way much greater than the velocity at the inlet, so we don't have to add on the velocity at state six to its energy. Since the mass flow rates at state six and seven were the same, they can be canceled out and now we can apply cold air standard assumptions and its isentropic efficiency.

$$C_p(T_6 - T_{7s}) = \frac{1}{2000} * {v_{7s}}^2$$

In this equation we still have two unknowns which would be the velocity at seven and its ideal temperature. To find this, we must leverage the properties of the air in the atmosphere. This means acknowledging that the pressure of the atmospheric air is the same throughout so P7=Pa. Using ideal isentropic formulas, we can determine the ideal temperature at state seven allowing us to find the ideal velocity at the end. The final step would be to apply its isentropic efficiency.

$$\left(\frac{T_{7s}}{T_6}\right) = \left(\frac{P_7}{P_6}\right)^{\frac{1-k}{k}}; \quad \eta_{HN} = \frac{{v_7}^2}{{v_{7s}}^2}$$

Cold Nozzle:

This is the second end component of the engine. It follows the same formulas that we used for the hot nozzle but this time it's between states two and eight.

$$\dot{m}_c(h_2) = \dot{m}_c\left(h_{8s} + \frac{1}{2000} * {v_{8s}}^2\right); \quad assumption: v_2 \ll v_8$$

$$C_p(T_2 - T_{8s}) = \frac{1}{2000} * {v_{8s}}^2$$

$$\left(\frac{T_{8s}}{T_2}\right) = \left(\frac{P_8}{P_2}\right)^{\frac{1-k}{k}}; \quad \eta_{HN} = \frac{{v_8}^2}{{v_{8s}}^2}$$

Mass Flow rates, Combustion Rate of Heat, and Efficiency:

Now that we have all the properties at each of the states, we can plug the values into the following equations to solve for the mass flow rates, rate of heat transferred in the combustion chamber, and the efficiency of the engine. With these formulas, we can use MATLAB to determine the best variables that will produce the most efficient engine.

$$[F_{th} = \dot{m}_h(v_7 - v_a) + \dot{m}_c(v_8 - v_a)] * \frac{1}{\dot{m}_h}$$

$$\frac{F_{th}}{\dot{m}_h} = (v_7 - v_a) + \frac{\dot{m}_c}{\dot{m}_h}(v_8 - v_a) \rightarrow \frac{\dot{m}_c}{\dot{m}_h} = Back\ pressure\ ratio$$

$$\dot{m}_c + \dot{m}_h = \dot{m}$$

$$\dot{Q}_{in} = \dot{m}_h(h_4 - h_3) \rightarrow \dot{Q}_{in} = \dot{m}_h C_p(T_4 - T_3)$$

$$\eta_{engine} = \frac{F_{th} * v_a}{\dot{Q}_{in}}$$

Results:

After using an iteration method on the variables that I chose to change such as the overall pressure ratio, back pressure ratio, the temperature at state four, and the fan pressure ratio, the results for the engine that produced the best efficiency at the desired altitude are in table 1.

Table 1: Results of the engine given my UIN number at altitude.

Calculations	Values
Thermal Efficiency	21.91%
Rate of heat transfer in Comb. Chamber	299010 KW
Mass flow rates ($\dot{m}_c, \dot{m}_h$)	(1824.6, 364.927) kg/s

The formulas as to how I solved these values can be found in the theory section of the report. Table 2 shows the pressures and temperatures at states a-8 and table 3 has the parameters that I used to create my engine.

Table 2:Properties for each state of the engine when plane is at altitude.

State	Pressure(kPa)	Temperature(K)
Atmospheric	22.7	216.8
One	23.3751	242.6449
Two	56.1003	318.4241
Three	818.1288	734.7115
Four	818.1288	1550
Five	244.3969	1133.7
Six	35.845	679.0376
Seven	22.7	597.6083
Eight	22.7	248.0643

Table 3:Parameters for engine at altitude.

Parameters	Values
Overall Pressure Ratio	35
Fan Pressure Ratio	2.4
Back Pressure Ratio	5
Temperature at State Four	1550
Isentropic efficiencies (ALL MAX)	[0.99,0.91,0.88,0.92,0.95,0.98,0.97]

Not only did were we asked to find the properties of each state at the altitude but we also made calculations from takeoff and if you can imagine the airplane engines on but the plane is not moving our efficiency becomes really bad, which you can see in table 4 and 5 has the values of pressure and temperature for each state. New parameters were used to keep engine properties realistic which can be seen in table 6.

Table 4:Results of the engine given my UIN number at takeoff.

Calculations	Values
Thermal Efficiency	0%
Rate of heat transfer in Comb. Chamber	86420 KW
Mass flow rates ($\dot{m}_c, \dot{m}_h$)	(669.2800, 133.8560) kg/s

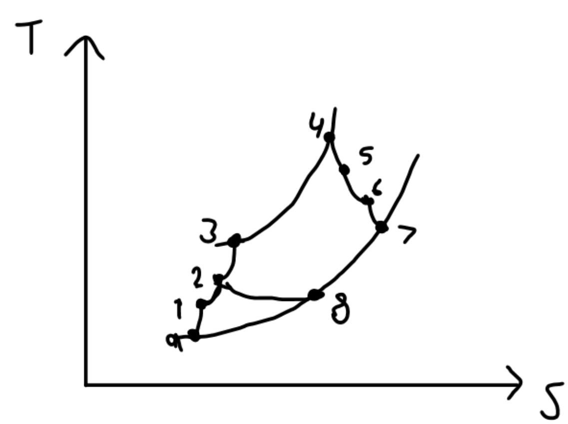
Table 5:Properties for each state of the engine when plane is at takeoff.

State	Pressure(kPa)	Temperature(K)
Atmospheric	100	298
One	100	301.01
Two	140	334.3883
Three	3500	907.5925
Four	3500	1550
Five	578.9052	976.7958
Six	247.2104	776.5264
Seven	100	603.1247
Eight	100	304.6583

Table 6:Parameters for engine at takeoff.

Parameters	Values
Overall Pressure Ratio	35
Fan Pressure Ratio	1.4
Back Pressure Ratio	5
Temperature at State Four	1550
Isentropic efficiencies (ALL MAX)	[0.99,0.91,0.88,0.92,0.95,0.98,0.97]

The final representation of the data that we found which can be shown in a visual manner is the T-S diagram of the engine and just for the sake of what model will be used for the main purpose of the engine, we will only create the diagram of the engine for when it is at altitude.



Discussion:

After compiling all the results and performing all the calculations, I would say that my data was pretty understandable. Given the range of parameters, there is really only so much efficiency that we could get out of the engine. Also given that one of our parameters was our UIN, it kind of made it luck as to whether or not someone gets a high efficiency. For example, if you had a bigger number UIN, then your efficiency would most likely be greater than someone else with a smaller number UIN and that's just because of the thermal efficiency formula. One issue I did happen to run in was the fact that some parameters, especially at takeoff, caused some of my answers to become imaginary. This most likely had to the with the fact that the atmospheric pressure was pretty high so after the turbines, my pressure at state six ended up being way smaller than the outside which is by nature impossible given the nozzle requires an inlet pressure greater than the exit, otherwise, air would be coming inside the engine. To fix this, I changed the fan pressure ratio to the lowest that it could be and it would also be important to note that the fan pressure ratio can vary will the engine is working, it does not have to be set at a certain ratio forever. Engineers have created a way to vary the pressure based on the surrounding conditions causing irregular calculations like the one I ran into, to never occur. For my isentropic efficiencies, I set them all to the max that they could be. I wanted the best efficiency and what better way to do that than to set the components to work in their best state. Also, after many iterations between altitude and takeoff, I came to the conclusion that having a high overall pressure ratio, high temperature at state four, and low back pressure ratio, gives the best results. As stated previously, the fan pressure ratio is the one varying parameter that will be chosen based on the atmospheric pressure.

As for how valid my work is, I would consider most of the formulas in order with the first law of thermodynamics. All components followed the first law calculations and because of the setup of this system, special equations had to be created in order to relate certain components in the system which was seen in the theory section of the report. It is, however, important to note that this is mostly all theory. While the calculations are in order and make sense with nature, it's most likely true that this scenario is not one hundred percent to what happens in reality. We assumed certain properties with the nozzle component that may have eliminated a small yet precise measurement that could be needed later on. We also used cold standard air assumptions to better help with all the calculations. As you can see from the derivation of the formulas that we used, all were in terms of temperature and that was only because of this assumption. In reality that is never the case. The last thing would be that the thrust was based off our UINs so that may also not be as valid as it should be. I would say that the calculations made can be applied if what is needed can be tolerated to a decent amount. After all, a plane crash is something that we would never want as an engineer.

Appendix:

MATLAB code:

```
%% Turbofan Project
clear;clc

Cp=1.005; %specific heat for air
K=1.4;

Pa=100; %Atmospheric pressure
Ta=298;%Atmospheric temp
Va=0; %Inlet Velocity
DAR=1.3; %Diffuser area ratio
MAX=0;

%Diffuser
E=100;
T1sguess=300;
while E>0.000000001
T1snew=(((Va^2)-((1/DAR)^2*(Ta/T1sguess)^(2/(K-1))*(Va^2)))/(2000*Cp))+Ta;
E=abs((T1snew-T1sguess)/T1snew);
T1sguess=T1snew;
end
T1s=T1snew;

IEDIFF=0.99; %(MAX IE)
T1=T1s/IEDIFF;
P1=(T1s/Ta)^((K-1)/K)*Pa;

%Variables
Rp=35;
RpFAN=1.4;
BPR=5;
T4=1550;

RpCOMP=Rp/RpFAN;

%Fan
IEFAN=0.91; %VARIABLE (MAX)
T2s=(RpFAN)^((K-1)/K)*T1; %Ideal temperature at state 2
T2=(T2s-T1)/(IEFAN)+T1;
P2=P1*RpFAN;

%High pressure compressor
IECOMP=0.88; %VARIABLE (MAX)
T3s=(RpCOMP)^((K-1)/K)*T2; %Ideal temperature at state 3
T3=(T3s-T2)/(IECOMP)+T2;
P3=Rp*P1;

%Combustion chamber
IEHPT=0.92; %VARIABLE(MAX)
P4=P3;

%High pressure turbine
```

```
T5=T4-(T3-T2);
T5s=T4-((T4-T5)/IEHPT);
P5 = P4*((T5s/T4)^(K/(K-1)));% Finding pressure with ideal conditions

%Low pressure turbine
IELPT=0.95;
T6=T5-((BPR+1)*(T2-T1));
T6s=T5-((T5-T6)/IELPT);
P6 = P5*((T6s/T5)^(K/(K-1)));

%Hot nozzle
P7=Pa;
IEHN=0.98; %(MAX IE)
T7s=T6*((P7/P6).^((K-1)/(K)));
T7=T6-IEHN*(T6-T7s);
V7s=sqrt(2000*Cp*(T6-T7s));
V7=sqrt(IEHN*(V7s^2));

%Cold nozzle
P8=Pa;
IECN=0.97; %(MAX IE)
T8s=T2*((P8/P2)^((K-1)/K));
T8=T2-IECN*(T2-T8s);
V8s=sqrt(2000*Cp*(T2-T8s));
V8=sqrt(IECN*(V8s^2));


%Mass flow rates
UIN=48526475;
FTH=UIN/200;
MH=FTH/((V7-Va)+BPR*(V8-Va));
MC=MH*BPR;
MOVERALL=MC+MH;

%Rate of heat transfer in CC
QIN=MH*Cp*(T4-T3);

%Thermal Efficiency
TE=(FTH*Va)/(1000*QIN);
```