

Hydraulics

Lab #1

Ana Paula Lopez
Diego Munoz
Pierre Henry
Christian Arriaga-Franco

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Abstract

In this laboratory, three fundamental hydraulic concepts were introduced through different experiment stations: pneumatic cylinders, pipe friction losses, and centrifugal pumps. The pneumatic cylinder station demonstrated extension and retraction forces and how these forces varied with supply pressure, with results applied to an uncertainty analysis. Pressure losses in smooth and rough pipes were measured to calculate the Darcy friction factor and head loss. The centrifugal pump station provided data under different flow conditions to determine pump head and efficiency. Overall, the lab enhanced understanding of measurement challenges and experimental uncertainty in fluid systems.

Objective

The objectives for this lab are for us to familiarize ourselves with how to measure with dial calipers, understand the importance of uncertainty, and study various components dealing with fluids. Sure, you can take a measurement and report that singular measurement as the final measurement but is that the true and correct measurement. Most likely not, and through this lab we can use methods such as sampling to better justify an answer for the true measurement. The concepts that we are going to use our methods of measuring on will be for hydraulics. Through hydraulics, we should be able to determine force outputs in both retracting and extending scenarios for given pressures and surface areas. The concepts that we studied through the various components include pipe flow and causes for changes in head for centrifugal pumps. Through the pipe flow, we should be able to find any losses in energy due to friction from a rough pipes surface versus a smooth pipes surface. By changing the flow rate for the centrifugal pump, we should note any changes in the head and efficiency.

Introduction

Understanding fluids is important in many aspects because of how we may transfer and use fluids. The fluids themselves can be used like water for agricultural purposes, and we must find out ways to transport that fluid which is where understanding the theory behind how pumps work becomes important. We also may use fluids as part of mechanisms to help supply forces because of how compressible they are. Overall, using fluids helps provide safer, cleaner, and more efficient designs. In this lab, we get to see how fluids are used inside of a piston cylinder device to generate a force in both extending and retracting scenarios. Through the stations in the lab, we can also see how the properties of a fluid that is transferred may be dependent on factors such as friction or flow.

To elaborate more on how we can view these fluids perform real life functions, we must first understand the theory behind it all. Starting with pistons and cylinders, the system consists of a compressor which supplies the air into our pressure regulator and from here we can control the pressure going into the piston.

As you can see in figure 1, air is flowing in through the side with the rod, meaning that the piston is retracting but if we change the flows with the valves we are provided, we can make the air go through the side with the cap end, therefore extending the rod.

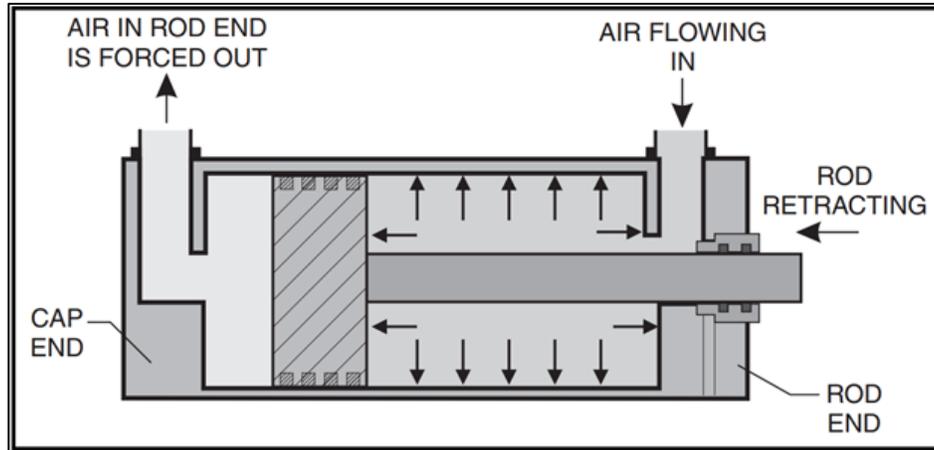


Figure 1: Anatomy of the pneumatic cylinder.

Theoretically the piston should exert more force when extending and the formulas explaining why are below:

$$F = P \cdot A \quad \mathbf{1a.}$$

$$A_{cap} = \frac{\pi}{4} D^2 \quad \mathbf{1b.}$$

$$A_{rod} = \frac{\pi}{4} (D^2 - d^2) \quad \mathbf{1c.}$$

Equation 1a is the general form for how we calculate the force exerted by the piston and normally the piston would have pressure exerting on both sides of the piston so we would have a subtractive force acting against the piston. In this case, the pressure on the other side of the piston we are putting air into is completely gone so there is no pressure acting against it, meaning we only have an additive force. With that, equation 1b is the area for which the pressure is acting on when extending, so the air is pushing on the cap area, and equation 1c is the area for which the pressure is acting on when retracting, so the air is pushing on the cap area minus the rod area. Since there is less area to push on when retracting, the force will be smaller. When performing in the lab, we should experience all these trends, verifying the theory is correct.

Another part of this section is also being able to present and justify any data that is collected. This is exclusively for any uncertainties in our measured values. The most influential form of error from this experiment would be our measurements from the dial calipers and pressure recorded from the pressure regulator. Since these two values are what the force depends on, we'll use equations for propagation to obtain our final value of uncertainty for the calculated force which are shown in equation 2a-2e. What we measure from the weight scale should be in this range.

$$u_f = \sqrt{\left(\frac{\partial F}{\partial P} u_P\right)^2 + \left(\frac{\partial F}{\partial A} u_A\right)^2} \quad \mathbf{2a.}$$

$$\frac{\partial F}{\partial P} = A \quad \text{and} \quad \frac{\partial F}{\partial A} = P \quad \mathbf{2b/2c.}$$

$$u_D = S \cdot t_{p,n-1} \quad \mathbf{2d.}$$

$$u_A = \frac{\pi}{2} D \cdot u_D \quad \mathbf{2e.}$$

Next is to study how friction may have an effect on our fluid, water, while flowing through the pipe. The friction we see here in fluids does have a negative effect on the water that is traveling by reducing the pressure which reduces the head, or the maximum height the water can travel. Having a rough surface along the wall would dissipate some of the energy it contains as it goes through turbulence leaving the pressure at the end less than it was at the beginning of the pipe. With the Darcy-Weisbach equation, we can find the friction factor and calculate the loss in head shown below:

$$\frac{\Delta P}{L} = f_D \frac{\rho v^2}{2D_h} \quad \mathbf{3a.}$$

$$h_{loss} = f_D \frac{Lv^2}{D2g} \quad \mathbf{3b.}$$

All the changes in pressure with their corresponding flow rates for equation 3a and 3b are found in the lab but calculation is required to find the friction factor and head loss, using the corresponding equations.

The last component we studied was a centrifugal water pump. What we want to understand in this part of the lab is how the components' performance may change with the properties of the given fluid, water. In this pump, as the flow rate increases, the head of the pump will decrease and so will the efficiency. This happens due to an increase in friction, turbulence, and recirculation of water as it flows through the pump. The formulas we can use to quantify the efficiency and head are shown in equation 4a and 4b.

$$H = \frac{P_d}{\rho g} \quad \mathbf{4a.}$$

$$\eta_H = \frac{Q \cdot \Delta P}{T \cdot \omega} \quad \mathbf{4b.}$$

The discharge pressure, inlet pressure, torque, pump angular velocity, and flow rate are all measured from the device to fill out these equations.

Procedure

The procedure of how the lab is carried out is relatively simple and should not cause any trouble, but all steps must be followed correctly to ensure accuracy. The order in which we carried out the lab is piston, pipe flow, then pump. This order does not have to be followed exactly but the procedures within each section do.

Starting with the piston, the first step is to have each group member measure both the cylinder inside diameter and rod diameter two times with the dial calipers. Once that is complete, the system can now be set up. The connection follows this order: air from the compressor, hose, pressure regulator, and gated valve system which is already connected to the piston (air from compressor should be turned off). After everything is connected and the valves are both switched off, the air can be turned on, and the pressure within the regulator will rise. From here the rest of the experiment can be carried out. Choose a pressure of 40psi on the pressure regulator, set up the scale for the valve which will be opened, tare the scale, and open

the valve. A weight on the scale should be displayed and that will be the force. Close the valve and switch the scale to the other side to perform retracting or extending depending on what was started first with. Do this with pressures ranging from 40-5 in increments of 5 for both retracting and extending.

Next is pipe flow. The pipe flow experiment station consisted of a variable flow rate pump with a water basin, smooth and rough pipe sections secured for testing, and a digital manometer. The pipe sections featured quick release connectors for connection to the pump inlet and outlet hoses as well as a manometer port at each end of the two sections. To begin the experiment, grab digital calipers and measure the distance between the pressure points. Starting with the smooth pipe, make sure both the inlet tube and outlet tube are connected to the smooth pipe as well as the digital manometer. With everything connected, turn on the machine by pressing the green button. Now starting at a flow rate close to the minimum, record the change in pressure across the pipe section, and continue this by increasing the flow rate measuring the change in pressure at each of the flow rates until all 5 spots in the table are filled. To ensure the best accuracy, obtain a variety of flow rates across the pump range. After all measurements for the smooth pipe are recorded, set the flow rate back to the bottom and turn the machine off. Switch the inlet, outlet, and manometer to the rough pipe and repeat the steps that were taken for the smooth pipe.

The last station is the centrifugal pump which should first be checked to see if all the correct valves are open or closed. Look at the picture provided for the best guide. Once all the valves are in their correct positions, turn the control panel on with the switch in the bottom right. Next, turn the pump on with the small switch to the left of the nob, Now, turn the nob and the rpm will increase. Get this as close to 3000 so the flow rate can be varied. Once the RPM is as close to 3000 as possible, the flow rate on the right side of the tank is changed to 1000 L/hr. It may seem like it's at 1000, but this setting really means 0 L/hr. In a sequential order, set the flow rate, measure inlet pressure, discharge pressure, and torque. These should all vary with flow rate. Once the values are measured for 0, increase in flow rate from 1000 to 6000 in

increments of 1000 and take measurements. Remember, the flow rate will always be 1000 less than what it says.

Results and Discussion

Section 1 – Pneumatic Cylinder

To better understand the trends and behavior for the pneumatic cylinders, we can perform calculations and plot graphs to create visuals and verify theory. The calculations for the pneumatic cylinder included those of force and area. With measured values such as the diameter of the cylinder cap and rod we were able to first calculate any expected theoretical values for the force that the piston must exert, this includes any calculated retracting and extending forces. An uncertainty value was also calculated for this force value which ideally would contain the measured value we obtain from the force scale. The first step to calculating the force is to know the area that we are applying pressure to. For this, all area values are calculated using the average or mean of the measurements and utilizing equations 1b and 1c.

Table 1: Extending and retracting area for pneumatic cylinder.

(units)	Cap Area	Rod Area	Extending Area	Retracting Area
mm ²	288.399	31.135	288.399	257.264
in ²	-	-	0.447	0.399

Since the unit for pressure was taken as psi in the lab, a conversion factor of 0.0393701 in/mm was applied for the area in the table to calculate the force the pneumatic cylinder is expected to produce since the force is pressure times area as shown in equation 1a.

Table 2: Calculated extending and retracting force for given pressures.

Pressure (psi)	Calculated Extending Force (lbs)	Calculated Retracting Force (lbs)
5	2.235101537	1.9938018
10	4.470203074	3.987603599
15	6.705304611	5.981405399
20	8.940406148	7.975207199
25	11.17550769	9.969008999
30	13.41060922	11.9628108
35	15.64571076	13.9566126

40	17.8808123	15.9504144
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These values may be precise, but they are not accurate, while measuring the diameter and pressure to make the force calculation, there was some uncertainty that needs to get applied to these force values in order to justify any varying values of force that we measure which will most likely not be the same number as the calculated ones. To perform this, we will use equations 2a-2e. Equation 2d requires the standard deviation for the cap diameter as well as the t-value given a 95% probability. The t-value can be found in a t-table while the standard deviation was calculated in excel and the formula will be shown in the appendix.

Table 3: Standard deviation and mean for measured diameter values.

-	Standard Deviation	Mean
Diameter (mm)	0.0243	19.1625

Table 4: Calculated uncertainty values for diameter, area, and pressure.

-	Diameter (mm)	Area (mm ² /in ²)	Pressure (psi) - chosen
Uncertainty	0.05758	1.73332/0.002687	2

The same conversion factor that was applied to the area earlier was also applied to its uncertainty and now we can apply all these numbers to equation 2a at each varying pressure to get their respective uncertainty values and just to compare and discuss, graphs will be made on these final values.

Table 5: All measured and calculated values for extending and retracting forces w/uncertainties.

Pressure(psi)	Uncertainty Force (lbs)	Extending Measured Force (lbs)	Extending Calculated Force (lbs)	Retracting Measured force (lbs)	Retracting Calculated Force (lbs)
5	0.894141529	3.21	2.235101537	2.26	1.9938018
10	0.894444202	3.94	4.470203074	3.31	3.987603599
15	0.89494843	6.4	6.705304611	5.66	5.981405399
20	0.895653872	8.44	8.940406148	7.23	7.975207199
25	0.896560053	10.92	11.17550769	9.07	9.969008999
30	0.897666366	12.99	13.41060922	10.95	11.9628108
35	0.898972072	15.7	15.64571076	13.46	13.9566126
40	0.900476302	16.94	17.8808123	14.51	15.9504144

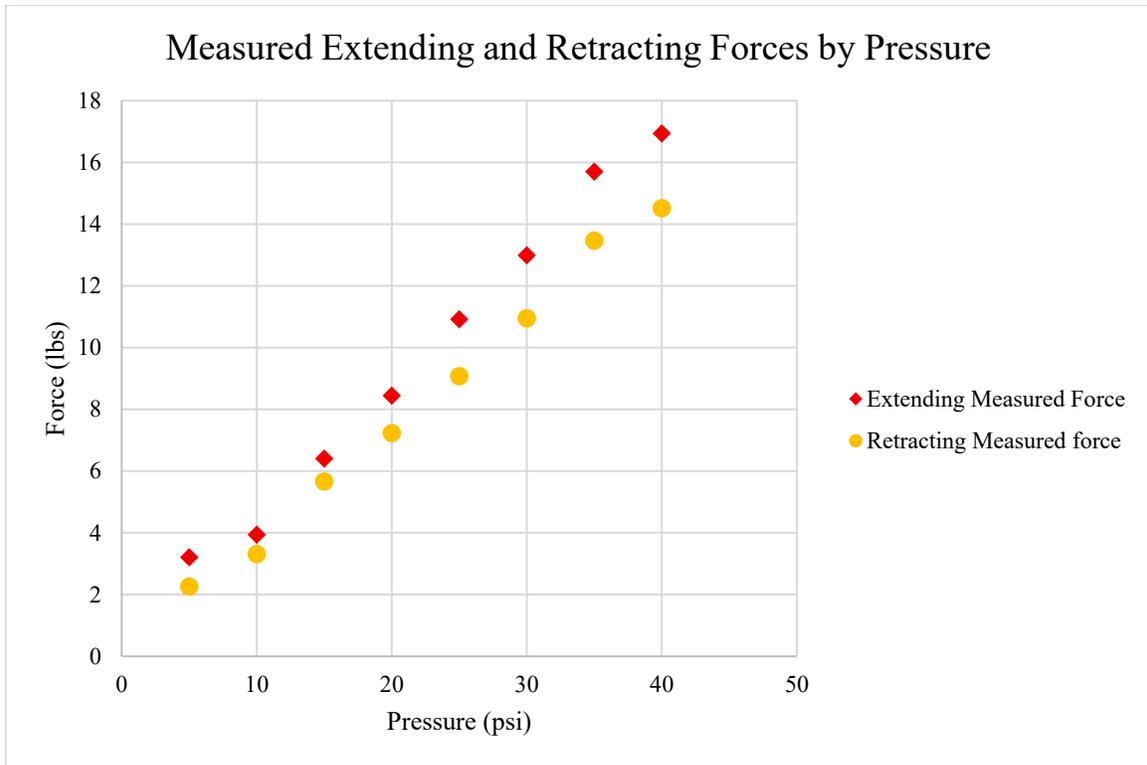


Figure 2: Graph of measured extending and retracting forces vs. pressure created in Microsoft Excel

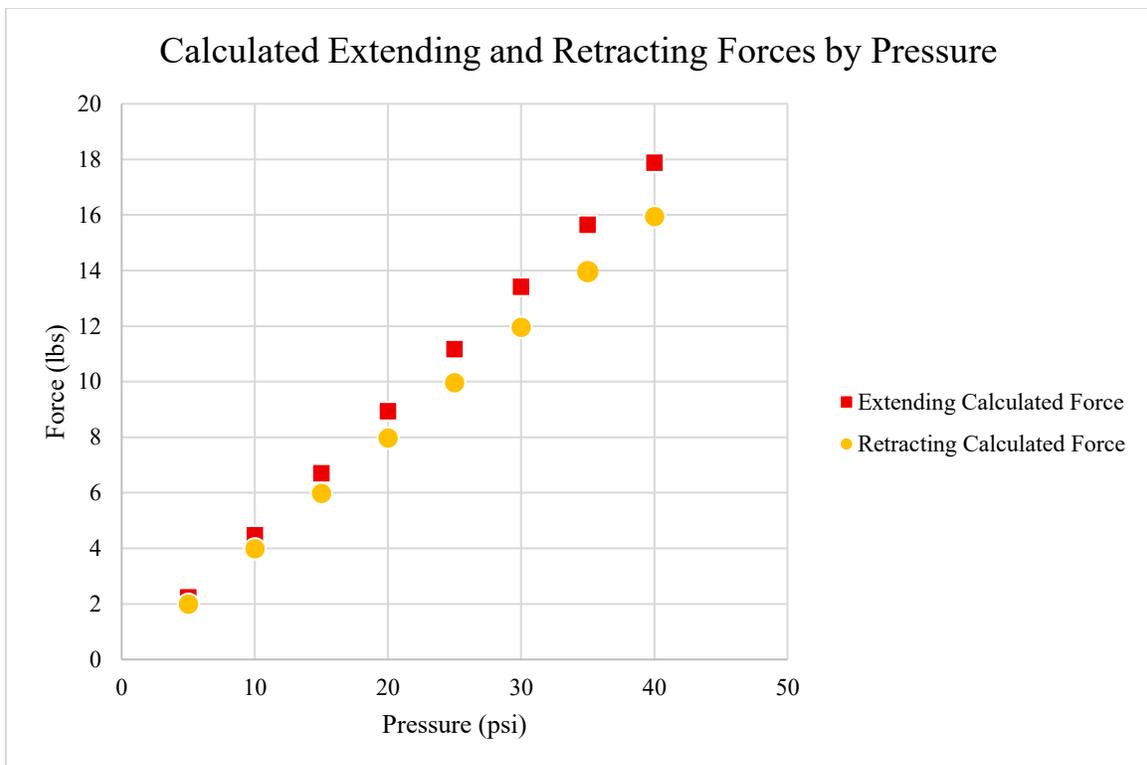


Figure 3: Graph of calculated extending and retracting forces vs. pressure created in Microsoft Excel

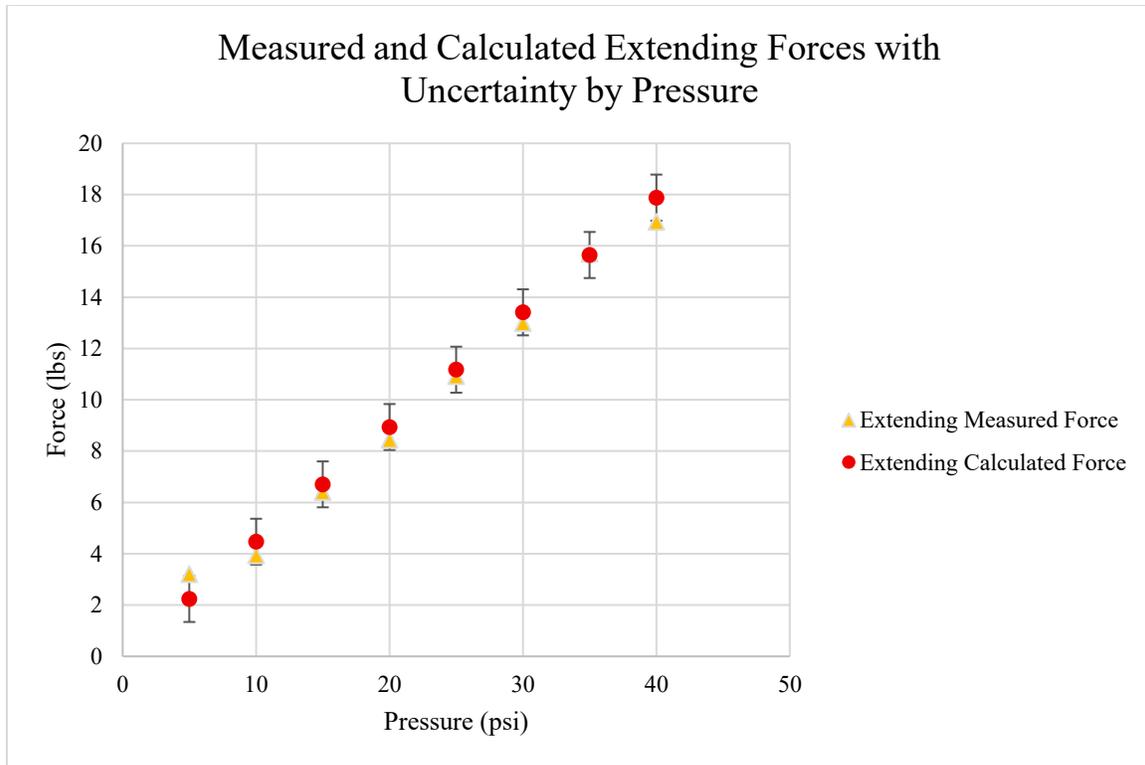


Figure 4: Graph of measured and calculated extending forces w/uncertainties vs pressure created in Microsoft Excel

With all calculations done and the visuals are made, we can see how well our measured values line up with the calculated theoretical values that we got. Figure 2 and 3 relatively follow the same shape where the force increases with pressure and that is expected because they are proportional in equation 1a and because of the fact that the area in extending force is greater, it also shows that trend on the graph where the retracting force is less than the extending force. This verifies a theoretical pattern that should occur. As for figure 4 which is all about uncertainties, we can see how well our measured values fit inside the ranges of uncertainty for the calculated values and for the most part, they do. The only two values not within the range are the first and last one and to best justify why that is would be because of how extreme the values are in the sense that we are using such a small pressure that the load scale wouldn't be able to pick up on any miniscule values which could be important. As for the last one, the pressure may be so high that the regulator has more trouble keeping the pressure at that level so the uncertainty for which we gave to the

pressure may have been greater than just 2psi. This value alone is an important factor to the uncertainty for the force so having the wrong one could have excluded our measured values. This also goes for the area which depends on the diameter and that was also measured. Although we did take 8 samples, that still may not be enough to get as close to the true diameter as possible. If this is off or even the standard deviation, these all play roles in whether or not our measured values are inside the acceptable range. Even though the range allows for some error, it's still questionable as to whether the range is the best range we could have taken because the fact is that our calculations have uncertainties so when we create them, its best to take the most reasonable and accurate measurements, otherwise we may not know if our measured values are truly within the range, just like we see in our two data points.

Section 2 – Pipe flow

Utilizing the experimental volume flow rates and pressure differentials recorded during the lab, the Darcy friction factor and head loss due to major losses were able to be calculated. Tables 11 and 12 contain the experimental data and necessary pipe dimensions from both the smooth and rough pipe samples. Tables 6 and 7, contain the calculated friction factor and head loss given the experimental data. The density of water was not included in the tables for simplicity and was assumed to be 1000 kg/m^3 . The flow velocity was derived using the volume flow rate and pipe cross-sectional area. All values were converted to standard SI units in Tables 6 and 7.

Table 6: Friction factor and head loss at varying flow rates calculated in Microsoft Excel – Smooth pipe

Volume Flow Rate (m^3/s)	Pressure Differential (Pa)	Distance Between Pressure Points (m)	Pipe Inner Diameter (m)	Friction Factor - Smooth	Head Loss - Smooth
1.6667E-05	100	0.069	0.006	0.0501	0.0102
3.3333E-05	300	0.069	0.006	0.0375	0.0306
5.0000E-05	600	0.069	0.006	0.0334	0.0612
5.8333E-05	900	0.069	0.006	0.0368	0.0917
6.6667E-05	1400	0.069	0.006	0.0438	0.1427

Table 7: Friction factor and head loss at varying flow rates calculated in Microsoft Excel – Rough pipe

Volume Flow Rate (m ³ /s)	Pressure Differential (Pa)	Distance Between Pressure Points (m)	Pipe Inner Diameter (m)	Friction Factor - Rough	Head Loss - Rough
1.6667E-05	1200	0.069	0.006	0.6006	0.1223
2.5000E-05	1800	0.069	0.006	0.4004	0.1835
3.3333E-05	2500	0.069	0.006	0.3128	0.2548
4.1667E-05	3700	0.069	0.006	0.2963	0.3772
5.0000E-05	5000	0.069	0.006	0.2781	0.5097

The calculated friction factor and head loss were plotted against volume flow rate in Figure 5 and Figure 6, respectively. Figure 5 plots the exponential relationship between volume flow rate and friction factor for the smooth and rough pipe samples. The exponential relationship is due to the friction factor being inversely proportional to the square of the fluid velocity which increases with volume flow rate, as noted in equation 3a. The comparatively higher friction factor for the rough sample was caused by the corrugated pipe profile.

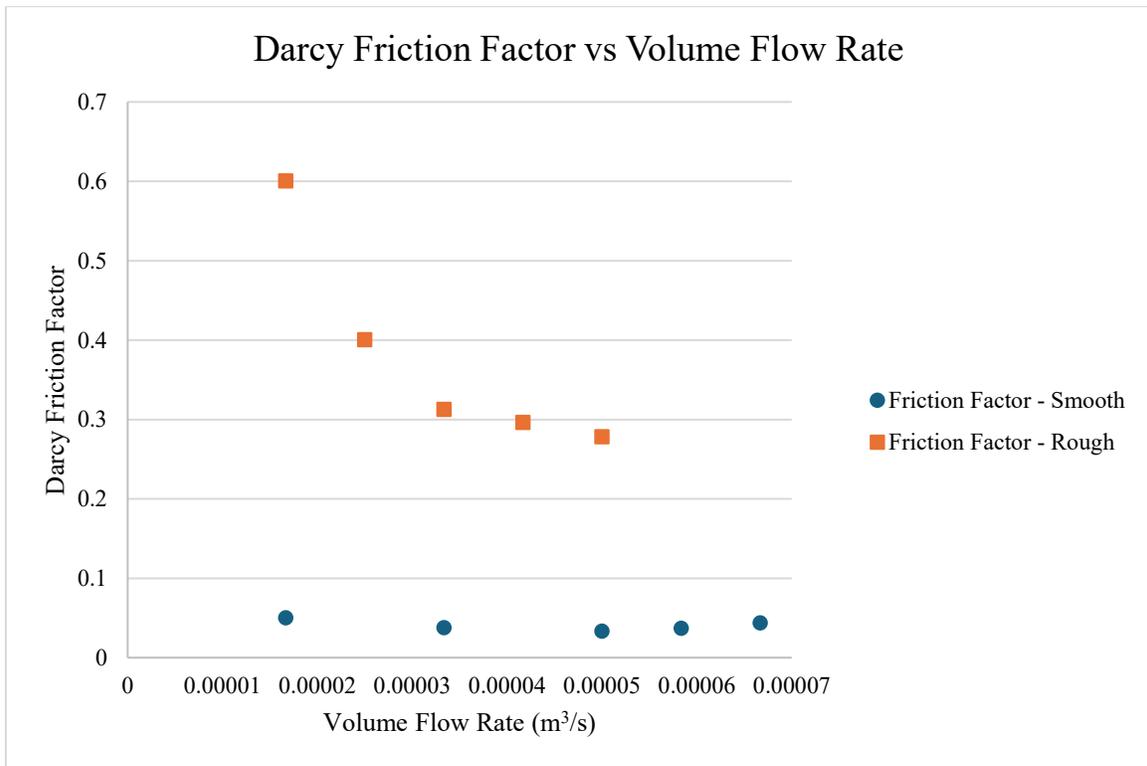


Figure 5: Plot of calculated friction factor vs experimental volume flow rate created in Microsoft Excel

Figure 6, plots the exponential relationship between head loss and volume flow rate. The exponential relationship is due to head loss being proportional to the square of the fluid velocity which increases with volume flow rate, as noted in equation 3b. The comparatively higher head loss in the rough pipe section was caused by the corrugated pipe profile and the according losses and turbulence.

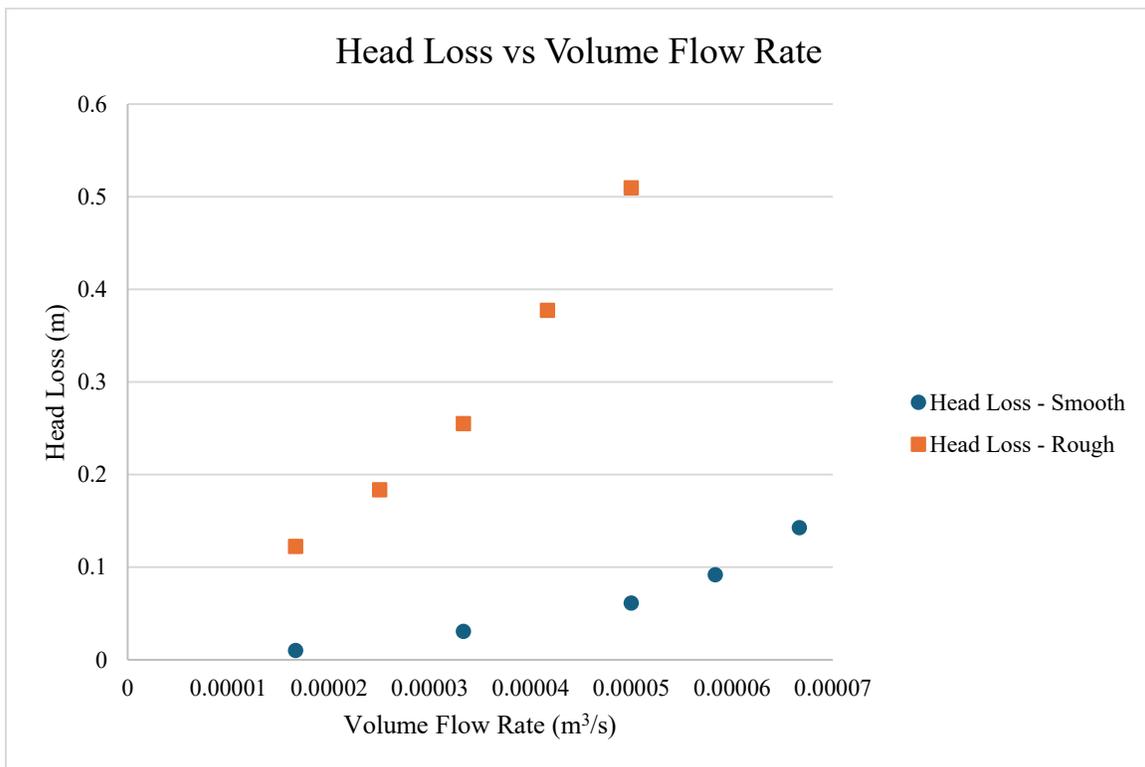


Figure 6: Plot of calculated head loss vs experimental volume flow rate created in Microsoft Excel

The way that losses are associated with pipe flow is analogous to the way resistance and current are related to voltage loss in a circuit. When a resistor is added to a circuit, there is an inherent voltage drop across that resistor proportional to the input voltage, current, and component resistance. In terms of pipe flow, the input pressure is treated as the input voltage, volume flow rate the current, and friction factor the resistance. Thus, in pipe flow the pressure drop along a section is analogous to the voltage drop across a component.

In Figure 5, it is shown that the friction factor decreases as the volume flow rate increases. This would be the same as saying the resistance would need to decrease as the current increases given a constant voltage source, as derived from Ohm's Law (eq. 5a). Thus, we understand the friction factor to be analogous to a resistor with a resistance that is a function of current.

In Figure 6, we see that the head loss increased with volume flow rate. This is similar to the way a component has a larger voltage drop as the current increases across a constant resistance component, given that the power source can supply additional voltage for the required current, as derived from Ohm's Law (eq. 5a). Thus, the pump should have been able to supply additional pressure to provide a constant current.

In terms of this study, we know that head loss increases and the friction factor decreases with increasing volume flow rate. However, head loss is directly proportional to the friction factor, but exponentially proportional to the flow velocity, so it is understood that greater losses are introduced with increased volume flow rate. Furthermore, Figure 6 builds on this principle, showing that increases in losses are exaggerated with the greater turbulence introduced by a rough pipe profile.

Section 3 – Centrifugal Pump

Centrifugal Pumps are typically used in water distribution systems, making it important to understand how different flow rates affect the pump head and how comparatively efficient the pump is. Calculating the pump head helps us determine how high the pump can move fluid within a system which is deduced by the ratio of the discharge pressure and the specific weight of the fluid. Table 11 presents the discharge pressure along with the density of the water to be multiplied by gravity to derive specific weight along with the calculated pump head. All units were converted to their standard SI units for easier calculations.

Table 8: Calculated pump head at varying flowrates

Volume Flow Rate (m ³ /s)	Discharge Pressure (Pa)	Fluid Density (kg/m ³)	Pump Head (m)
0	187000	1000	19.06218
2.7778E-04	177000	1000	18.04281
5.5556E-04	155000	1000	15.8002
8.3333E-04	124000	1000	12.64016
1.1111E-03	81000	1000	8.256881
1.1944E-03	46000	1000	4.689093

The relationship between the pump flow rate and pump head are distinctly non-linear as can be seen in Figure 7. The curve of the graph is downward concluding that an increase in flowrate decreases the height to which the pump can effectively push the fluid if the density speed stays constant (Table 10). It can be assumed that recording more data points would further decrease head until no fluid can be pushed at all.

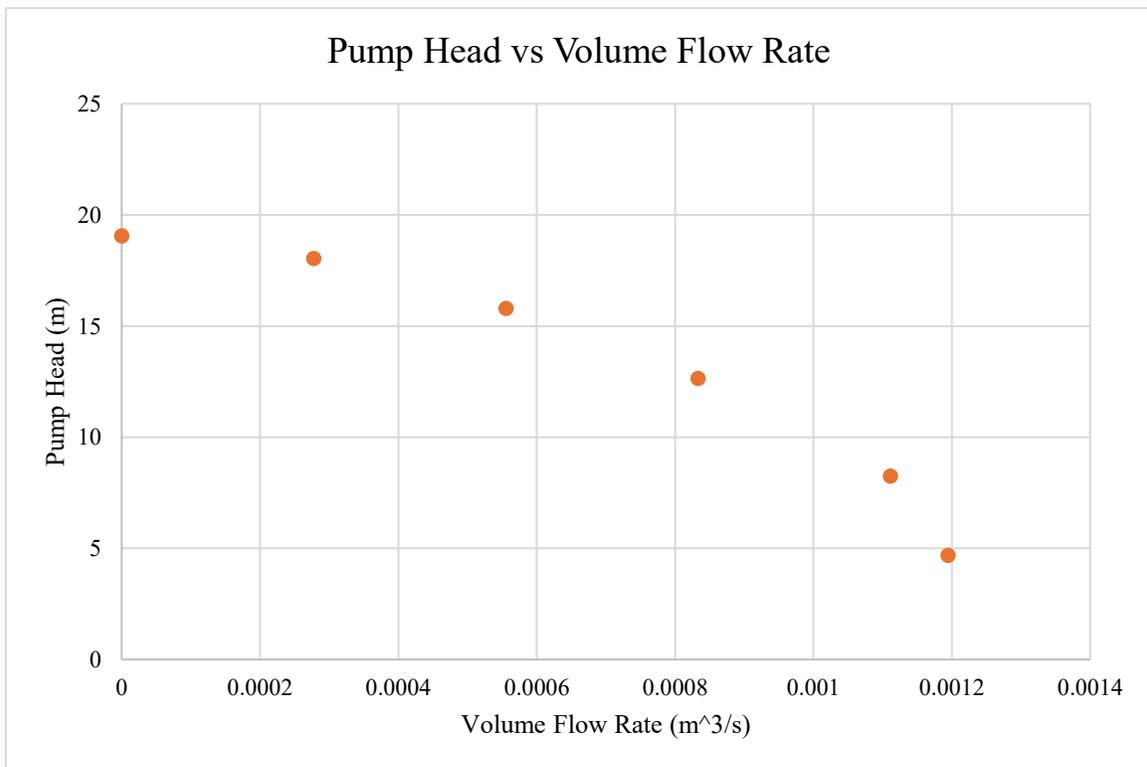


Figure 7: Plot of calculated pump head vs experimental volume flow rate created in Microsoft Excel

The mechanical efficiency of the pump is a ratio between the hydraulic power transmitted to the fluid over the mechanical power produced by the pump shaft. In Table 12, calculated pump efficiency is shown along

with the flow rate, pressure difference, torque, and angular velocity used in calculations. The speed in Table 10 is converted to angular velocity for simplicity in calculations as well as all other units converted to SI.

Table 9: Calculated pump mechanical efficiency at varying flowrates

Volume Flow Rate (m ³ /s)	Discharge and Inlet Pressure Difference (Pa)	Torque (N-m)	Angular velocity (rad/s)	Pump Mechanical Efficiency
0	184000	0.8	311.646	0
2.7778E-04	176000	0.9	311.646	1.74E-01
5.5556E-04	159000	1.06	311.646	2.67E-01
8.3333E-04	134000	1.17	311.646	3.06E-01
1.1111E-03	101000	1.26	311.646	2.86E-01
1.1944E-03	74000	1.28	311.646	2.22E-01

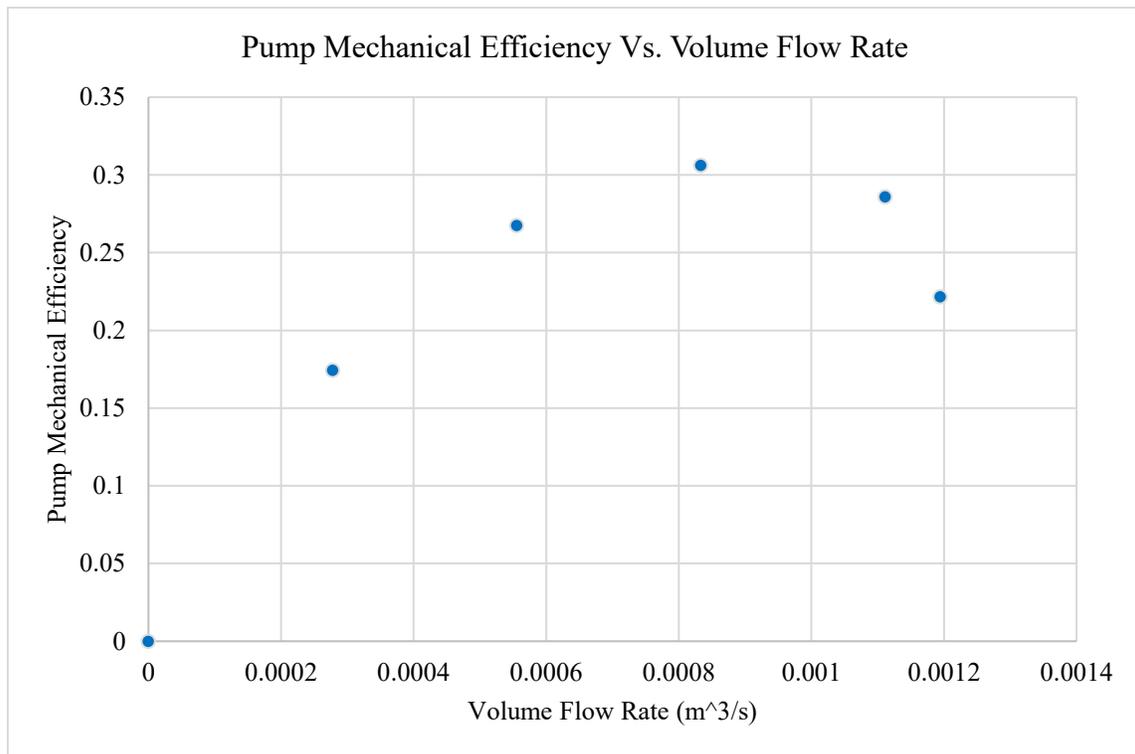


Figure 8: Plot of pump mechanical efficiency vs experimental volume flow rate created in Microsoft Excel

Now that the pump head and efficiency plotted, the relationship between the two can be better understood. In figure 8, the plotted efficiency peaks at the 4th data point with a steep decline afterwards. When choosing the specifications of a centrifugal pump it's important to first know what the purpose of it is in each system. The efficiency of the pump in water distribution systems household water delivery should be prioritized so

as not waste energy during the process. Comparing the most efficient point in Figure 8 to the head in Figure 7 with the same flowrate we say that we sacrificed a difference from the max height. Though, increasing the torque or the angular velocity would increase efficiency at a lower flow rate if required in reference to the mechanical efficiency equation (eq. 4b).

Conclusion/References

Section 1 – Pneumatic cylinder

As a result, from this section of the experiment we were able to verify all theory that was made on the pneumatic cylinder as well as get adjusted to presenting more confident data. Through the relationship between force, pressure, and area, we noticed trends that were in compliance with the graphs presented in the theoretical tests. In all trials for varying pressures, the pneumatic cylinders performed as expected, however, there were some forces which did not fall within the range of the calculated force values after all uncertainties were applied. It's not certain to assume that the pneumatic cylinder was the one that was faulty in this scenario because of how well it followed the trends as shown in figure 2 but instead, it's because we had so many uncertainties to apply to the calculated values that they themselves could be off. This includes measurements such as the rod diameter, cap diameter, and pressure which all had uncertainties. Also, there is the fact that we only obtained so little samples heavily affected the random error uncertainties, which play a big role in total uncertainty. Apart from verifying the theory behind pneumatic cylinders, this section also helps us develop an understanding of the various sources of error we should be aware of to better improve upon so we can justify any values that were measured and calculated.

Section 2 – Pipe Flow

The pipe flow experiment verified the theoretical relationship between friction factor and head loss. The Darcy–Weisbach equation (eq. 3a) related our independent variable, the volume flow rate, and the dependent variable, the friction factor. The experimental values showed that the velocity squared term was the controlling term, causing friction factor to decrease exponentially with increasing flow velocity. However, while the friction factor did decrease exponentially with increasing flow velocity, head loss increased exponentially with increasing flow velocity despite decreases in the friction factor as the velocity squared term was the controlling term. The results show that volume flow rate, and thus flow velocity given that the fluid is a liquid and treated as incompressible, has the greatest effect on head loss for a given pipe. The friction factor and head loss curves behaved similarly between the smooth and rough pipe sections,

however the rough pipe section had comparatively higher friction factors and head loss due to turbulence and fluid friction caused by the corrugated surface.

For real-world applications, these findings show that lower volume flow rates of liquids require exponentially lower initial pressures to achieve the same vertical fluid displacement with a significant dependence on the roughness of the pipe section. This conceptual understanding is useful in the development of pump systems for buildings, aquariums, industry, and more. Water towers act in a similar manner, where water is slowly filled into the reservoir, but is allowed to be used by consumers at higher rates during peak hours. In this case a lower pressure pump can be used to meet peak consumption demands as it keeps the reservoir steadily between its maximum and minimum capacity, creating a more efficient system.

Section 3 – Centrifugal Pump

Centrifugal pumps are very important in domestic applications such as home water distribution systems. It is important to understand the specifications such as the pump head and mechanical efficiency when designing these systems and how different elements within relate to one another. The pump head refers to the height to which the pump can push a fluid in a pipe. From the calculations we can see that as the flow rate increases the head non-linearly decreases. Though the obvious conclusion is to select a lower flow rate if a higher head output is desired, this is not entirely the case when considering the pump efficiency. When analyzing the curve of the pumps mechanical efficiency at the same flow rates and comparing it to the pump head we can conclude the efficiency does not follow the same path with a peak at the third flow rate data point. There are significant tradeoffs between the head and efficiency an engineer should be aware of when designing or picking a centrifugal pump to use in each application.

Appendix/Questions

$$V = IR \quad \mathbf{5a.}$$

Section 1 – Raw Data and uncertainty Sample Calculations*Table 10: Table of raw measured data of pneumatic cylinder dimensions for calculated force values*

Measurement Number (N)	Cylinder Inside Diameter (mm)	Cylinder Rod Diameter (mm)
1	19.18	6.33
2	19.2	6.31
3	19.14	6.28
4	19.16	6.33
5	19.13	6.22
6	19.17	6.23
7	19.18	6.39
8	19.14	6.28

$$\bar{x}_{ID} = \frac{\sum x_i}{N} = \frac{19.18 + 19.2 + \dots}{8} = 19.1625 \text{ mm}$$

$$s_{ID} = \sqrt{\frac{\sum (x_i - \bar{x}_{ID})^2}{N - 1}} = \sqrt{\frac{(19.18 - 19.1625)^2 + (19.2 - 19.1625)^2 + \dots}{7}} \approx 0.02435 \text{ mm}$$

$$\bar{x}_{RD} = \frac{\sum x_i}{N} = \frac{6.33 + 6.31 + \dots}{8} = 6.29625 \text{ mm}$$

*Standard deviation for rod diameter – since we did not find uncertainties for retracting forces, we do not need to apply any uncertainties found in the rod diameter, which includes calculations involving standard deviation.

Force Sample Calculation (Extending – 5psi)

$$F = P * A * \text{Conversion Factor} = \left(\frac{5 \text{ lbs}}{\text{in}^2}\right) \left(\frac{\pi * (19.1625 \text{ mm})^2}{4}\right) \left(\frac{0.0393701 \text{ in}}{\text{mm}}\right)^2 \approx 2.235 \text{ lbs}$$

Force Sample Calculation (Retracting – 5psi)

$$F = P * A * \text{Conversion Factor}$$

$$= \left(\frac{5 \text{ lbs}}{\text{in}^2} \right) \left(\frac{\pi * ((19.1625 \text{ mm})^2 - (6.29625 \text{ mm})^2)}{4} \right) \left(\frac{0.0393701 \text{ in}}{\text{mm}} \right)^2 \approx 1.994 \text{ lbs}$$

Extending force uncertainty propagation:

$$F = P * A$$

Need both uncertainty for pressure and area. The uncertainty for pressure was chosen to be 2psi for the remaining calculations.

$$u_F = \sqrt{(u_{F,P})^2 + (u_{F,A})^2} = \sqrt{\left(\frac{\partial F}{\partial P} * u_P \right)^2 + \left(\frac{\partial F}{\partial A} * u_A \right)^2}$$

$$\frac{\partial F}{\partial P} = A ; \frac{\partial F}{\partial A} = P$$

$$u_F = \sqrt{(A * u_P)^2 + (P * u_A)^2}$$

The area in this equation remains constant by using the sample mean area found but since the pressure changes for every iteration, so will the uncertainty. Previously stated, the uncertainty for pressure was chosen to be 2 psi, $u_p = 2 \text{ psi}$, now the area uncertainty must be calculated.

Area uncertainty propagation:

$$A = \frac{\pi}{4} D^2 \rightarrow u_A = \frac{dA}{dD} \Big|_{D = \bar{x}_{ID}} * u_D$$

$$\text{Inner diameter uncertainty} \rightarrow u_D = s_{ID} * T_{p,n-1} = (0.02435)(2.365) \approx 0.05758 \text{ mm} \quad (95\%)$$

$$u_A = \frac{\pi(\bar{x}_{ID})}{2} * u_D = \frac{\pi(19.1625)}{2} * 0.05758 \approx 1.733 \text{ mm}^2$$

Force Uncertainty Sample Calculation (Extending – 5psi)

$$u_F = \sqrt{(A * \text{Conversion Factor} * u_P)^2 + (P * u_A * \text{Conversion Factor})^2}$$

$$u_{F,5psi} = \sqrt{\left(\left(\frac{\pi * (19.1625 \text{ mm})^2}{4}\right)\left(\frac{0.0393701 \text{ in}}{\text{mm}}\right)^2\left(\frac{2 \text{ lbs}}{\text{in}^2}\right)\right)^2 + \left(\left(\frac{5 \text{ lbs}}{\text{in}^2}\right)(1.733 \text{ mm}^2)\left(\frac{0.0393701 \text{ in}}{\text{mm}}\right)^2\right)^2}$$

$$\approx 0.894142 \text{ lbs} \rightarrow F_{5 \text{ psi, ext}} = 2.235 \pm 0.894142$$

Section 2 – Raw Data and uncertainty Sample Calculations

Table 11: Experimental pipe flow pressure differential at varying flow rates – Smooth Pipe

Volume Flow Rate (L/min)	1.0	2.0	3.0	3.5	4.0	Distance Between Pressure Points (mm)	Inner Diameter (mm)
Pressure Difference (KPa)	0.1	0.3	0.6	0.9	1.4	69.00	6

Table 12: Experimental pipe flow pressure differential at varying flow rates – Rough Pipe

Volume Flow Rate (L/min)	1.0	1.5	2.0	2.5	3.0	Distance Between Pressure Points (mm)	Inner Diameter (mm)
Pressure Difference (KPa)	1.2	1.8	2.5	3.7	5.0	69.00	6

Friction Factor Sample Calculation (Table 11, Q = 3 L/min)

$$3.0 \text{ L/min} = 5.0 \times 10^{-5} \text{ m}^3/\text{s}$$

$$A_{\text{pipe}} = \pi \left(\frac{0.006}{2}\right)^2 [\text{m}^2] = 2.82743 \times 10^{-5} \text{ m}^2$$

$$v = \frac{Q}{A} = \frac{5.0 \times 10^{-5}}{2.82743 \times 10^{-5}} [\text{m/s}] = 1.76839 \text{ m/s}$$

$$\rho = 1000 \text{ kg/m}^3$$

$$D_h = \frac{4(A_{\text{pipe}})}{\pi D} = \frac{4(2.82743 \times 10^{-5})}{\pi(0.006)} [\text{m}] = 0.006 \text{ m}$$

$$f_D = \frac{2\Delta P D_h}{L \rho v^2} = \frac{2(0.6 \times 10^3)(0.006)}{0.069(1000)(1.76839)^2} = 0.03337$$

Head Loss Sample Calculation (Table 11, Q = 3 L/min)

$$h_{\text{loss}} = f_D \frac{L v^2}{D 2g} = 0.0334 \left(\frac{0.069(1.76839)^2}{0.006(2(9.81))}\right) [\text{m}] = 0.0612 \text{ m}$$

Section 3 – Raw Data and uncertainty Sample Calculations

Table 13: Centrifugal pump speed, torque, discharge and inlet pressure at varying flow rates

	Volume Flow Rate (L/Hr)	Speed (rpm)	Torque (N-m)	Discharge Pressure (Bar)	Inlet Pressure (Bar)
1	0	2976	0.8	1.87	0.03
2	1000	2976	0.9	1.77	0.01
3	2000	2976	1.06	1.55	-0.04
4	3000	2976	1.17	1.24	-0.10
5	4000	2976	1.26	0.81	-0.20
6	4300	2976	1.28	0.46	-0.28

Pump Head Sample Calculation

$$H = \frac{P_d}{\rho g} = \frac{177000 \text{ Pa}}{1000 \frac{\text{kg}}{\text{m}^3} * 9.81 \frac{\text{m}}{\text{s}^2}} = 18.04281 \text{ m}$$

Pump Mechanical Efficiency Sample Calculation

$$\eta_H = \frac{N_H}{N_S} = \frac{Q * \Delta P}{T * \omega} = \frac{\left(2.7778 * 10^{-4} \frac{\text{m}^3}{\text{s}}\right) * 176000 \text{ Pa}}{0.9 \text{ Nm} * 311.646 \frac{\text{rad}}{\text{s}}} = 1.74 * 10^{-1}$$

$$\Delta P = 177000 \text{ Pa} - 1000 \text{ Pa} = 176000 \text{ Pa}$$

$$\omega = 2976 \text{ rpm} * \frac{38 \text{ rev}}{1 \text{ min}} * \frac{1 \text{ min}}{60 \text{ s}} * \frac{2\pi}{1} = 311.646 \text{ rad/s}$$